Exercise 61

Find the absolute maximum and absolute minimum values of f on the given interval.

$$f(x) = \ln(x^2 + x + 1), \quad [-1, 1]$$

Solution

Take the derivative of the function.

$$\begin{aligned} f'(x) &= \frac{d}{dx} \ln(x^2 + x + 1) \\ &= \frac{1}{x^2 + x + 1} \cdot \frac{d}{dx}(x^2 + x + 1) \\ &= \frac{1}{x^2 + x + 1} \cdot (2x + 1) \\ &= \frac{2x + 1}{x^2 + x + 1} \end{aligned}$$

Set what's in the numerator equal to zero, and set what's in the denominator equal to zero. Solve each equation for x.

$$\begin{aligned} 2x + 1 &= 0 \\ x &= -\frac{1}{2} \\ x &= -\frac{1}{2} \end{aligned} \qquad \begin{aligned} x^2 + x + 1 &= 0 \\ x &= \frac{-1 \pm \sqrt{1^2 - 4(1)(1)}}{2} \\ x &= \frac{-1 \pm i\sqrt{3}}{2} \end{aligned}$$

$x = -1/2$ is within $[-1, 1]$, so evaluate f here.

$$f\left(-\frac{1}{2}\right) = \ln\left[\left(-\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right) + 1\right] = -\ln\frac{4}{3} \approx -0.287682 \qquad \text{(absolute minimum)}$$

Now evaluate the function at the endpoints of the interval.

$$f(-1) = \ln[(-1)^2 + (-1) + 1] = 0$$

$$f(1) = \ln[(1)^2 + (1) + 1] = \ln 3 \approx 1.09861 \qquad \text{(absolute maximum)}$$

The smallest and largest of these numbers are the absolute minimum and maximum, respectively, over the interval $[-1, 1]$.

The graph of the function below illustrates these results.